Chapter 13

CHAIN PRICE AND VOLUME AGGREGATES FOR THE SYSTEM OF NATIONAL ACCOUNTS

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1. Summary and Introduction

This paper constitutes a critique of the recommendations for changing the System of National Accounts 1968 (SNA68) contained in the Systems of National Accounts 1993 (SNA93) on volume measures of gross domestic product (GDP). These recommendations are contained in Chapter XVI of the SNA93, authored by the distinguished English economist Peter Hill. Basically, this paper endorses the SNA93 recommendation for annual chain linking, but not its support for chain Fisher aggregates, nor, as a second-best solution, chain Laspeyres aggregates. I argue it is both feasible and desirable to calculate chain fixed-price aggregates that do not have the dangerous propensity to chain drift exhibited by chain Laspeyres aggregates. And these fixed-price aggregates can be calculated as direct series for the most recent period and so be additive over commodities, industries or regions, unlike their chain Fisher counterparts.

Perhaps the best way to summarize the present paper is to list Hill's five recommendations (H1-H5), followed, one by one, by my proposed amendments (B1-B5):

(H1) Original: *The preferred measure of year-to-year movements of GDP volume is a Fisher volume index; changes over longer periods being obtained by chaining; i.e., by cumulating the year-to-year movements.*

(B1) Amended: The preferred measure of year-to-year movements of GDP volume is an Edgeworth-Marshall[2] volume aggregate (see formula (4-1) below) changes over longer periods being obtained by chaining; i.e., by cumulating the year-to-year movements.

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[2] I have referred to index formulae by their inventors. For formulae with multiple inventors, they are listed in alphabetical order without any attempt to assign a primacy among them. Thus, Edgeworth-Marshall instead of Edgeworth or Marshall-Edgeworth, Montgomery-Vartia instead of VartiaI, Sato-Vartia instead of VartiaII, and Bowley-Sidgwick instead of Sidgwick. Bowley was actually a propogandist rather than a discoverer of the Bowley-Sidgwick formula, but he should share the credit since Sidgwick did little more than note the possibility of an

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(H2) Original: *The preferred measure of year-to-year inflation for GDP is, therefore, a Fisher price index; price changes over longer periods being obtained by chaining the year-to-year price movements: the measurement of inflation is accorded equal priority with the volume movements*.

(B2) Amended: The preferred measure of year-to-year inflation for GDP is an Edgeworth-Marshall price index (see formula (7-1) below).

However, as the Edgeworth-Marshall formula does not satisfy the strong factor reversal test, direct and chain implicit price indexes should also be calculated and published. The direct implicit price index should be based on the ratio of the expenditure series at current prices to the expenditure series at constant prices (i.e., the weighted average of prices over two years).

For highly cyclical commodities, a modification of the Edgeworth-Marshall formula will be required for the inflation indicator, and the basket reference period can span three to five years as required. (This may also be necessary for the cyclical components of the GDP volume measure.) To accommodate seasonal commodities, the price index could incorporate a seasonal weighting pattern, preferably using the Rothwell or Balk formula.

(H3) Original: *Chain indexes that use Laspeyres volume indices to measure year-to-year movements in the volume of GDP and Paasche price indices to measure year-to-year inflation provide acceptable alternatives to Fisher indices.*

(B3) Amended: Countries too small or too poor to implement annual-link chain measures should calculate chain Laspeyres volume aggregates, with rebasing of these every five years, the base year of each five-year span being its central year. In addition to the chain Paasche price indexes that would be the counterpart of these chain volume aggregates, they should also calculate chain Laspeyres price indexes which would also be rebased every five years, with the central year of the five-year span serving as the basket reference year.

(H4) Original: *The chain indices for total final expenditures, imports and GDP cannot be additively consistent whichever formula is used, but this need not prevent time series of values being compiled by extrapolating base year values by the appropriate chain indices.*

(B4) Amended: The chain volume aggregates for total final expenditures, imports and GDP will continue to be additive because they will be linked backward and not forward.

(H5) *Original: Chain indices should only be used to measure year-to-year movements and not quarter to quarter movements.*

(B5) Amended: Chain volume aggregates should be calculated both annually and quarterly (or monthly). The fixed-price structure of the volume aggregates (i.e., using average prices over two years) will apply to both annual and quarterly (or monthly) series, ensuring that meaningful measures of quarterly (or monthly) volume change can be derived for all consecutive quarters (or months) for the direct series, and for all but the Q4-to-Q1 movements (December-to-January movements) of the chain series. Similarly, chain volume price indexes should be calculated both annually and quarterly (or monthly). The fixed-basket structure of the price indexes (i.e. using a two-year basket) will apply to both annual and quarterly (or monthly) series, ensuring that

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arithmetic cross between the Laspeyres and Paasche formulae, whereas Bowley somewhat advocated it. Auer (2004) and others refer to the Bowley-Sidgwick formula as the Drobisch formula, but I follow Diewert (1993) in attributing it to these two English-speaking economists. However, the Fisher formula remains the Fisher formula out of deference to established custom, even though others, including Bowley, wrote about it before he did.

meaningful measures of quarterly (or monthly) price change can be derived for all consecutive quarters (or months) for the direct series, and for all but the Q4-to-Q1 movements (December-to-January movements) of the chain series.

However direct volume aggregates (fixed-basket price indexes) would also be calculated over longer periods of up to 10-11 years, and linked backward to form chain series whose base prices (basket) would change every five years, with the base period (basket) always representing the central years of the five-year span. These series would provide meaningful measures of quarterly or month change when the chain measures failed to do so.

The most controversial of the proposed amendments may well be (A1) and (A2). Some economists would reject the Edgeworth-Marshall formula because it is not exact for any aggregator function, i.e. it is not a "superlative" formula, like the Fisher formula. If the choice of a formula must be limited to those defined as "superlative", then (A1) and (A2) should be rewritten to replace the Edgeworth-Marshall formula with the linear Walsh formula[3]. The Walsh formula is exact for the Generalized Linear aggregator function. Because of its matrix consistency properties, it would still be a better choice than the Fisher formula. This has been suggested by Diewert (1996) in his critique of Hill's paper.

However, the Edgeworth-Marshall formula would seem to be the better choice as, unlike the Walsh formula (or the Fisher formula), it respects the property of transactions equality (i.e., the importance of a transaction in the formula does not depend on the period in which it occurs.) Also, unlike the Walsh formula, it does not discard commodities from a volume aggregate if the price goes to zero from a positive price or vice-versa, nor does it discard commodities from a price index if the quantity goes to zero from a positive quantity or vice-versa.

2. The SNA68 Volume Measures

In the 1968 System of National Accounts (SNA68) the prescribed volume measures are Laspeyres volume aggregates. These are direct measures over the recent period, but linked backward prior to the base year. Thus additive consistency is preserved for the most recent history of the series, but not prior to the base year.

Eight features of the direct Laspeyres volume aggregates are worthy of note.

<u>First</u>, they are expenditure totals and not index numbers, with the formula:

$$\sum p_0 q_t \,.$$

Thus, they are Laspeyres volume aggregates, rather than Laspeyres volume indexes. (Here and elsewhere, summation is assumed to be over commodities unless otherwise indicated.)

The Laspeyres volume aggregates can be defined as measuring period t expenditures at base year 0 prices. When these aggregates are indexed to base year expenditures, the result is a Laspeyres volume index given by

[3] Hereafter this is referred to simply as the Walsh formula; there is also a logarithmic Walsh formula, but it is not discussed in this paper.

$$Q^{L}_{t/0} = \sum p_0 q_t / \sum p_0 q_0 .$$

In actual statistical practice, it is more common to publish expenditure series at constant prices than Laspeyres volume index values.[4]

Second, for each commodity, the base year price is defined as a unit value ($\bar{p}_0$) so that when multiplied by the corresponding quarterly quantities for the base year and aggregated over quarters ($q = 1, \ldots, 4$), the total will equal base year expenditure; i.e.,

$\sum_q p_{0q} q_{0q} = \sum_q \bar{p}_0 q_{0q}$, which requires that

(2-0) $$\bar{p}_0 = \sum_q p_{0q} q_{0q} / \sum_q q_{0q} = \sum_q p_{0q} \left(q_{0q} / \sum_q q_{0q} \right).$$

Thus, the base price is calculated as the unit value of all base year transactions, which is a weighted arithmetic mean of quarterly prices with quarterly quantities used as the weights. This is the uniquely optimal estimate of the average price for any homogeneous commodity, a point that should be underlined.[5]

It can be seen that if the production of a commodity were completely inelastic with respect to price, all of the quantities in (2.0) would be equal and the unit value would reduce to an arithmetic mean of prices, which always exceeds the geometric mean of prices.

As a mean of quarterly base prices weighted by quantities, the base year unit value can also be interpreted as a harmonic mean of prices weighted by expenditures; that is:

(2.1) $\bar{p}_0 = 1 / \sum_q w_{0q} (1/p_{0q})$, where $w_{0j} = p_{0q} q_{0q} / \sum_q p_{0q} q_{0q} = v_{0q} / \sum_q v_{0q}$.

In (2.1), $v_{0q} = p_{0q} q_{0q}$ denotes the transactions value. It can be seen that if production of the commodity were unit-elastic with respect to price, so that whatever the change in price, the same revenues were generated, the base year unit value would reduce to a simple harmonic mean of quarterly base prices, which is always less than the geometric mean of base prices. Note, in particular, that the unit value is not consistently higher or lower than the geometric mean; it is higher for commodities with higher price elasticities and lower for commodities with lower price elasticities.

[4] Until recently, the only important volume index that used to be published by the Canadian System of National Accounts (CSNA) was the index of industrial production. Even for this series, a chain volume aggregate, rather than a volume index, is now published instead.

[5] Suppose that exactly the same transactions occurred two years in a row at the same set of prices. The same amount of money would be required for these expenditures in both years. Suppose further that the budget for year 1 is established as the number of units purchased multiplied by the average price in year 0. If one chose an average price less than the unit value, the budget would be inadequate to make all the transactions required. If one chose a price greater than the unit value, then the budgeted funds would be more than needed. The resulting surplus would be greater the larger the discrepancy between the unit value and the overestimated annual price. The same would be true if there were an increase in the volume of transactions but the seasonal profile of volumes and prices remained constant.

In actual national accounting practice, one would rarely have actual price and quantity data to work with. Usually one would be dealing with value series and independently derived price indexes that serve as deflators. In this case, what would replace (2.1) is:

(2.2) $$\sum_q v_{0q} = \sum_q v_{0q} / P_{0q/0},$$

where $P_{0q/0}$ is the price index for a given quarter of year 0 with base year 0. Generally, the identity in (2.2) does not hold and it must be forced; i.e., the quarterly values for the base year must be prorated so that the value for quarter q is given by

(2.3) $$f \times v_{0q} / P_{0q/0}$$

where $f = \sum_q v_{0q} / (\sum_q v_{0q} / P_{0q/0})$.

(Strictly speaking, the adjustment factor *f* should be applied to all quarterly values, but in practice it is usually applied only to the base year quarterly values, creating a discontinuity between the base year estimates and the estimates to follow that is presumed to be slight.)

Third, the ratio of the expenditures at current prices to the expenditures at constant prices provides an implicit price index that has the Paasche formula:

$$P^P_{t/0} = \sum p_t q_t / \sum p_0 q_t .$$

This can serve as a price indicator for GDP. In production, Paasche price indexes offer official statisticians the choice of calculating volume aggregates directly or by deflating using the Paasche deflator. More specifically, volume aggregates have usually been calculated indirectly as seasonally adjusted series at current prices deflated by raw or seasonally adjusted Paasche price indexes.

However, the importance of this association of Laspeyres volume aggregates with Paasche price indexes should not be exaggerated. A Paasche price index is a poor indicator of price change except for binary or two-period comparisons since quarterly price changes are always distorted by changes from one basket to another. In Canada, dissatisfaction with the Paasche deflators led to the development of Laspeyres price measures, and then to the use of chain Laspeyres series, years before the current chain volume measures were introduced.

Fourth, there is additivity of the components of GDP in the volume aggregates, as one would expect, since they are simply the expenditures of each quarter at a common set of prices. That is, expenditures on consumer goods and services sum to total consumer expenditure, construction expenditures and investment in machinery and equipment sum to gross fixed capital formation, and so forth.

There is also additivity of the monthly or quarterly GDP estimates, which sum to equal the annual estimates, and the same applies for all major components of GDP.

Finally, there is at least the possibility with this kind of structure of having additivity of provincial or regional GDP estimates to the national total. In Canada, this was imposed. Hence, the provincial GDP estimates by industry at 1997 prices sum to the all-Canada totals. This multi-dimensional additivity over commodities, industries, months or quarters, and regions can be characterized as *matrix consistency*. Thus, a desirable feature of GDP volume aggregates is additivity along any dimension of a matrix of values at constant prices.

Fifth, the change between any two quarters or any two years of the volume aggregates satisfies the *proportionality test*: i.e., if all of the quantities in a given period are k times the corresponding quantities in the comparison period, then the volume aggregate shows a k-fold increase between the two periods. As a special case of this, the volume aggregates satisfy the identity test. Hence, if all the quantities in a given period are identical with the corresponding quantities in the comparison period, then the level of the volume aggregate will be the same in the given period as in the comparison period. (The identity test is just the proportionality test for k=1.) This is a consequence of all expenditures for all periods being converted to the same set of fixed prices.

Sixth, the Laspeyres volume aggregates are strictly or strongly consistent in aggregation; that is, they can be equally well calculated in a single stage from basic components, or in two stages by first using the basic components and then using higher level subaggregates, with the same formula applied at both stages (see also the appendix).

Seventh, the Laspeyres volume aggregates have the new commodity property alluded to by Irving Fisher in *The Making of Index Numbers*:

> "The introduction of a new commodity ought, evidently, to change, in some degree, any price index which pretends to be a sensitive expression of the data from which it is computed (*unless,* of course, *the new commodity happens to have a price relative exactly equal to the index number*)" [emphasis added].[6]

If a new commodity is added to the volume aggregate with the identical volume movements as the aggregate, the movement of the aggregate will not change. Although this seems like a banal property, it is quite useful in actual statistical work since frequently one may have weighting information of some kind for a new commodity for a reference year, but lack detailed price or quantity data. It is possible to create a volume series for a new commodity, explicitly or implicitly by imputing the group movement in a straightforward way. Of course, given detailed data for a new commodity, one would immediately know if it would push the aggregate series up or drag it down. This is quite important for agencies concerned with making their GDP by industry and GDP by expenditure estimates mesh, where the implementation of a methodology change in a given revision cycle could depend on whether it tended to increase or decrease an aggregate growth rate, and decisions are often made under severe time pressure.

Eighth, SNA68 recommended rebasing of the volume aggregates every five or ten years. Although the manual is vague about the mechanics involved, it clearly favoured retaining the same movements of historical series when the relative price structure was updated. It also favoured, if not quite so clearly, calculating volume aggregates at new base year prices starting from the base year itself, which became the Canadian practice for all rebasings that followed the publication of the SNA68 manual until the CSNA moved to chain Fisher estimates in 2001. For example, the 1961 base year was introduced for the period 1961 and after in the second quarter of 1969. Note that although quarterly estimates for the base year would be at base year prices, the annual benchmark estimates would reflect the previous base year. In terms of the previous

[6] Quoted by Krtscha (1984, p.136).

example, while CSNA calculated a quarterly series at 1961 prices from 1961 forward, the movement of the annual volume series for 1961 was based on 1957 prices. It should be noted that until the publication of the SNA68 manual, the CSNA had always applied the new base earlier than the base year itself, the 1949 base being applied starting in 1947 and the 1957 base starting in 1956.

Introducing a new base year only in the following year, in terms of the annual movement, certainly reduces the magnitude of the revisions of the volume aggregate series, but generally at a heavy price in terms of representativeness. Even with updatings of the base year every five years, the finalized estimates for the last year of a span will be five years removed from their reference year,.

The historical volume aggregates would be linked to the new volume aggregates using a rebasing factor for each series as follows:

$$(\sum p_5 q_5 / \sum p_0 q_5) \times \sum p_0 q_t ; \quad t = 0,1,\ldots,4 .$$

Hence expenditures at year 0 prices are linked to the new series at year 5 prices. Typically, there would be a loss of additivity when this rebasing was carried out for the historical period (e.g., the sum of expenditures on consumer services and consumer goods would no longer equal total consumer expenditure). In Canada, this was handled with an elaborate set of adjusting entries that re-establish additivity between the sum of components and their aggregate, although it would probably have been simpler just to publish a note of warning that due to chain linking, additivity did not hold.[7]

3. Major Weaknesses of the SNA68 Measures

The principal weakness of the SNA68 methodology was obvious to its framers even at the time it was promulgated. In a rapidly changing world, a set of constant prices can get out of date even after a five-year interval, and will be unrepresentative of the economy whose output it is supposed to measure. Thus, SNA68-type estimates frequently lack the quality of representativeness. Also, given a negative correlation between prices and quantities, there will tend to be a positive bias in direct Laspeyres volume measures, and this bias will be more serious the longer a single set of constant prices is retained. For this reason, those who decided on the procedures for the SNA68 considered the possibility of chaining volume series every year instead of every five or ten years, but finally rejected the idea for general application because of its onerous data requirements.

The SNA68 manual did give lukewarm support to the idea of calculating Fisher index numbers for the years between the new base year and the old base year, but this was probably never done in any country and certainly not in Canada. The problem was that, for example, between 1961 and 1971, two successive base years of the CSNA, one could only calculate one true Fisher volume index number, for 1971 as compared to 1961. One could of course, calculate index numbers for the intervening years from 1962 to 1970, that would also reflect a geometric

[7] This was the practice for the quarterly estimates of GDP by expenditure category. There were never any adjusting entries published for the monthly estimates of GDP by industry.

mean of indexes at 1961 prices and indexes at 1971 prices, but it didn't make any sense to give 1961 prices and 1971 prices the same influence on volume movements in 1962 as compared to 1961, when the price structure in 1962 was so much more like that of 1961.

A second major weakness of SNA68 is that the Laspeyres formula does not pass the time reversal test; i.e., the Laspeyres quantity index for year t with base year 0 is not equal to the reciprocal of the Laspeyres quantity index for year t with base year t as required by the time reversal test; i.e.,

$$Q^L_{t/0} = \frac{\sum p_0 q_t}{\sum p_0 q_0} \neq 1/Q^L_{0/t} = 1/\frac{\sum p_t q_0}{\sum p_t q_t} = \frac{\sum p_t q_t}{\sum p_t q_0} = Q^P_{t/0}$$

In fact, the reciprocal of the Laspeyres quantity index is the Paasche volume index, which shows the dual nature of the two formulae, one being the complement of the other. Suppose that prices and quantities, after changing in year 1, revert to their previous year 0 values in year 2. A chain Laspeyres volume index with a link at year 1 would show:

$$\frac{\sum p_0 q_1}{\sum p_0 q_0} \times \frac{\sum p_1 q_2}{\sum p_1 q_1} = \frac{\sum p_0 q_1}{\sum p_0 q_0} \times \frac{\sum p_1 q_0}{\sum p_1 q_1} \neq 1$$

Generally one would expect prices and quantities to be negatively correlated. The import of this is that a chain Laspeyres volume index may be subject to *chain index drift*, tending to drift upward due to a negative correlation between prices and quantities even if the overall level of output between two years at the same point in the business cycle is identical. More frequent chaining will not help this situation, but only worsen it, by strengthening the negative correlation between the index weights and quantity relatives.

The time reversal test has been dismissed as a useful criterion for index number formulae because in our world time never does run in reverse, but for statisticians who are more interested in producing reliable indexes than in clever word play, the failure of the Laspeyres formula to pass the time reversal test constitutes a serious problem.

A third weakness of the SNA68 is that the recommended volume estimates would not pass the *transactions equality test,* which states that, "the relative importance of each transaction is dependent only on its magnitude."[8] This is quite obvious, since the relative importance of all commodities in the Laspeyres volume estimates would be based only on the expenditures of base year 0. Base year transactions would consequently have a considerably greater influence on the estimates than those of other years.

This is similar to the property of representativeness already discussed but is not identical with it. For example, if one calculated a 20-year output series with base prices equal to the average annual prices over the entire 20-year period, such estimates would pass the transactions equality test (with transactions from every month of every year determining the base prices). However, by taking them from all years, the base prices would be poorly representative of any particular year, especially (if relative prices were strongly trending) the initial and final years.

[8] See Dikhanov (1994, p.3).

This particular weakness has not bothered most national accountants. The emphasis has been on finding a base year that was a normal year with a representative price structure. For example, in a comparison between output levels in 1913 and 1918, 1913 constant prices would be preferred over 1918 constant prices, since 1913 was a normal peacetime year and 1918 was a war year.[9] However, experience has shown that it is not that easy to find a "normal" year to serve as base. In the Canadian System of National Accounts (CSNA), some of the recent base years have proven less than optimal, including 1981 and 1992 which were both recession years. The year 1981 was especially poor as it was also a year when relative prices of important commodities were much higher than usual, as were nominal interest rates..

The use of a single year's base prices is especially problematical for industries such as agriculture where relative prices are persistently volatile. (It was to avoid the difficulty in finding a single representative base year that 1935-1939 base prices were adopted by the CSNA for its initial set of output estimates at constant prices, but subsequently base prices were always taken from a single year.)

4. The Natural Remedies for These Weaknesses

In summary, the SNA68 measures had three major weaknesses: lack of representativeness, an upward bias due to the use of the Laspeyres formula, and failure to pass the transactions equality test. All of these weaknesses can be corrected by calculating annually-linked volume series, where each year's expenditures are evaluated at the average of a given and the previous year's prices. Technically speaking, this would involve calculating a chain Edgeworth-Marshall volume series rather than a direct Laspeyres volume series. To make it clear how the proposed measures would differ from the SNA68 measures as well as the Hill SNA93 proposals, Hill's eight points will be rewritten in detail to reflect my proposals.

First, the Laspeyres volume aggregates would be replaced with expenditures at constant prices for years 0 and 1 combined, denoted hereafter by

$$\sum \bar{p}_{01} q_t$$

The annual index for year 1 for the above series is an Edgeworth-Marshall volume index, given as:

$$\text{(4-1)} \quad Q^{EM}_{1/0} = \sum \bar{p}_{01} q_1 / \sum \bar{p}_{01} q_0 \,.$$

It can be readily seen that such an index, in contrast with the Laspeyres volume index, passes the time reversal test; i.e.,

$$Q^{EM}_{1/0} = \frac{\sum \bar{p}_{01} q_1}{\sum \bar{p}_{01} q_0} = 1 / Q^{EM}_{0/1} = 1 / \frac{\sum \bar{p}_{01} q_0}{\sum \bar{p}_{01} q_1}.$$

[9] See Bowley (1924, p.92) where this argument is made in the context of a price index, with the Laspeyres formula favoured over the Fisher.

Therefore, chain Edgeworth-Marshall indexes will be less subject to chain drift than chain Laspeyres indexes.

Second, the use of base period prices defined over two years rather than one provides a more stable reference than a single year's prices would. Commodity by commodity, the base period price is defined so that the following equality holds:

$$\sum_q p_{0q}q_{0q} + \sum_q p_{1q}q_{1q} = \sum_{y=0}^{1}\sum_q \bar{p}_{01}q_{yq}.$$

For this to be true, it must also be so that:

$$\text{(4-2)} \quad \bar{p}_{01} = (\sum_{y=0}^{1}\sum_q p_{yq}q_{yq})/(\sum_{y=0}^{1}\sum_q q_{yq}) = \sum_{y=0}^{1}\sum_q p_{yq}(q_{yq}/(\sum_{y=0}^{1}\sum_q q_{yq}));$$

that is, the base price must be calculated as the unit value for the two years 0 and 1, which is the mean of the quarterly prices using quantities as weights. As discussed in section 2, this is the only appropriate way to calculate the average price for a homogeneous product. This base price can also be interpreted as a harmonic mean of the quarterly prices, with the weighting based on expenditures; i.e.,

$$\bar{p}_{01} = 1/\sum_{y=0}^{1}\sum_q w_{yq}(1/p_{yq}) \text{ where } w_{yj} = p_{yq}q_{yj}/\sum_{y=0}^{1}\sum_q p_{yq}q_{yq}$$

As in the SNA68 case, if one were working with value series and independently derived price indexes rather than prices and quantities, the requirement would be somewhat different. For base prices defined over years 0 and 1, one would require that:

$$\text{(4-3)} \quad \sum_{y=0}^{1}\sum_q v_{yq} = \sum_{y=0}^{1}\sum_q v_{yq}/P_{yq/01},$$

where $P_{yq/01}$ is the price index for a given quarter with a two-year base period covering years 0 and 1. Generally, the identity in (4-3) does not hold. Instead, the quarterly values for the base period must be prorated so that the value for quarter q of year y at constant prices for years 0 and 1 will be equal to the following:

$$\text{(4-4)} \quad f \times v_{yq}/P_{0q/0},$$

where $f = (\sum_{y=0}^{1}\sum_q v_{yq})/(\sum_{y=0}^{1}\sum_q v_{yq}/P_{yq/01})$.

As will be seen in the seventh point, the official chain volume estimates are to be based on annual links, so for the finalized estimates there is no issue of whether the adjustment factor should be applied to years 0 and following, or only to years 0 and 1.

Close inspection will show that this calculation of base prices for a two-year base period is completely analogous with the calculation of base prices for a one-year base period in the SNA68. Thus, it is somewhat surprising that this option seems to have been ignored in the literature on chain volume measures. The principle of Ockham's razor would suggest that the problems that plagued the SNA68 world should be solved with the least possible deviation from it. Instead the proposed solutions have usually related to different formulae than the fixed-price volume formula, notably the SNA93 recommendation in favour of the Fisher formula. When interest has been expressed in a fixed price formula, it usually relates to base prices defined as another average of the prices of years 0 and 1. In particular, the Walsh volume index, defined as:

$$(4\text{-}5) \quad Q_{01}^{w} = \frac{\sum \bar{p}_{01}^{w} q_1}{\sum \bar{p}_{01}^{w} q_0}$$

$$\text{where } \bar{p}_{01}^{w} = \sqrt{\bar{p}_0 \times \bar{p}_1} = \sqrt{1/\left(\sum_q w_{0q}(1/p_{0q})\right) \times \left(\sum_q w_{1q}(1/p_{1q})\right)}$$

has received favourable attention, notably from Erwin Diewert (1996). Note that in the Walsh volume index, the base prices represent geometric means of the average annual prices for years 0 and 1, but since these are calculated as in the SNA68 system, the Walsh base prices represent an odd hybrid: the unweighted geometric means of weighted harmonic means.

Formula (4-5) does not satisfy the property of transactions equality between the two years 0 and1 because both years have about the same impact on the calculation of the average base prices even if the volume of transactions was much greater in one year than the other. (This is a real possibility for new commodities, outmoded commodities or highly cyclical ones, as discussed in section 8.)

If the price of a commodity were zero in either year using (4-5), the Walsh base price would be zero, and the commodity would have no influence on the estimate of volume change for the year. If there are no sales of the commodity in a given year one must impute a price for it anyway; it does not simply disappear from the calculation. The Edgeworth-Marshall base prices are much better behaved in this respect. At the limit, if all transactions were in year 0 and none in year 1, the unit value for years 0 and 1 would reduce to the unit value for year 0 only.

Third, the ratio of the expenditures at current prices to the expenditures at constant prices provides an implicit price index:

$$(4\text{-}6) \quad P_{1/01}^{EMIPI} = \frac{\sum p_1 q_1}{\sum \bar{p}_{01} q_0} \div \frac{\sum \bar{p}_{01} q_1}{\sum \bar{p}_{01} q_0} = \frac{\sum p_1 q_1}{\sum \bar{p}_{01} q_1}.$$

This is the Edgeworth-Marshall counterpart to the Paasche price index and would serve much the same purpose. Operationally, one can choose between calculating volume aggregates directly or by deflating value observations using implicit price indexes. However, like the Paasche price indexes, these Edgeworth-Marshall implicit price indexes would be poor indicators of price change in and of themselves, since all quarterly changes would be distorted by changes in the index basket from one quarter to the next.

Note that the first term after the first equal sign in (4-6) represents, not an index of values at current prices, as in a formula for a Paasche price index, but rather the index of values for the current year t to the base year values re-expressed in terms of average prices for years 0 and 1. Although this may look a little strange to someone used to the standard formulas, it is quite logical. If one rejects a single year's prices for making volume comparisons why would one wish to index current expenditures to base year expenditures at their own prices, rather than at a more normal set of prices?

Note also that while the counterpart of a Laspeyres volume index number with base year 0 is a Paasche price index number with the same base, the counterpart of an Edgeworth-Marshall volume index number with base year 0 is an implicit price index with a multi-year base covering years 0 and 1.

Fourth, as with the Laspeyres volume aggregates, these volume series would be additive across all important dimensions, and so would satisfy the property of matrix consistency. This point cannot be stressed enough, since the biggest drawback of SNA93's proposed Fisher aggregates is that they are not additive or matrix consistent.

Fifth, the change between any two quarters, or between the base year and the following year, of the Edgeworth-Marshall volume aggregates satisfies the *proportionality test*. This is because, like the Laspeyres aggregates, all expenditures for all periods are converted to the same set of fixed prices.

Sixth, the Edgeworth-Marshall volume aggregates are weakly consistent in aggregation; i.e., they can be equally well calculated in a single stage from basic components or in two stages, by first calculating subaggregates from basic components using the Edgeworth-Marshall formula, and then adding up the volume subaggregates to get the aggregate series. The Edgeworth-Marshall formula is weakly consistent in aggregation. At the second stage of aggregation the Edgeworth-Marshall formula would not be used to calculate the aggregate from the subaggregates; that would yield an incorrect result. In this respect, Edgeworth-Marshall aggregates differ from SNA68's Laspeyres volume measures, which were strongly consistent in aggregation. However, the Edgeworth-Marshall aggregates are superior to their Fisher aggregates which are not even weakly consistent in aggregation.[10]

Seventh, the Edgeworth-Marshall formula also has the new good property possessed by the Laspeyres formula. This is logical, since any formula that can be expressed as a weighted average of its component indexes will satisfy the new good property. It is one of the weaknesses of the Fisher formula that, being a geometric mean of the Laspeyres and Paasche indexes, it does not satisfy the new good property. This is obvious if one thinks of a new good being added for year 2004, with an imputed growth ratio equal to the Fisher volume index number for 2004 that was calculated without it. Since this Fisher index number will differ from its Laspeyres or Paasche components, both of these components will change when the new good is included, and so will the Fisher aggregate. To include the new good without it making a difference to the overall index, it would be necessary to treat the same new good as having two distinct movements: a movement equal to that of the Laspeyres measure for the Laspeyres calculation, and a movement equal to that of the Paasche measure for the Paasche calculation, so that, overall, the Fisher measure would remain unchanged. This would be possible, but would also be messy and error-prone.

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[10] The failure of the Fisher formula to be even weakly consistent in aggregation, is not, as is sometimes alleged, empirically insignificant; the erroneous calculation of a Fisher aggregate in two or more stages instead of in a single stage can even reverse the direction of measured growth of an aggregate.

Eighth, Edgeworth-Marshall volume aggregates would be calculated as a chain volume series whose base would change every year. This would also be true of the Edgeworth-Marshall price aggregate. For the current year the estimate would be given by:

$$(4\text{-}7) \quad \sum \bar{p}_{t-1t} q_t .$$

Earlier years would follow the recursion:

$$(4\text{-}8) \quad \frac{\sum \bar{p}_{t-1t} q_t}{\sum \bar{p}_{t-2t-1} q_t} \times \sum \bar{p}_{t-2t-1} q_{t-1} .$$

That is, the volume aggregate would always be linked backwards, so that the additivity of the most recent periods would be preserved. This is different from what is recommended in the SNA93 where chaining is forward and additivity is not preserved for the most recent volume estimates even if a Laspeyres formula rather than a Fisher formula is used to calculate them.

In an era of electronic publications, there is no great difficulty in changing the base of volume estimates every year as this proposal would require. This has been proven by the Australian and British National Accounts programs that have adopted backward linking for their annually-linked chain volume measures based on the Laspeyres formula. For the Statistics Canada monthly GDP by industry estimates on which the present author worked, this proposal would entail the publication of from 43 to 54 months of data that were additive over an annual production cycle with backward linking in place. The time span is so long due to delays in the updating of annual benchmark price and volume estimates.

These estimates would be representative in a way that the old volume estimates were not, since the base prices would be updated every year. They would also satisfy the requirement of transactions equality, since every year y would be used to calculate the base prices for two annual comparisons: y-1 to y and y to y+1, and the importance of year y in the pooled set of base prices would depend on the volume of transactions associated with year y.

This is in contrast with the Fisher case recommended by SNA93 where the principle of transactions equality is violated. Instead, each year's transactions have about the same influence on the measure of base prices regardless of their volume. As can be seen from (4-1), if the volume of transactions in year 0 is very small compared to year 1, the base prices for years 0-1 will essentially be year 1 base prices. In the Fisher case, by contrast, one would have:

$$(4\text{-}9) \quad Q^F_{1/0} = \left[\frac{\sum p_0 q_1}{\sum p_0 q_0} \frac{\sum p_1 q_1}{\sum p_1 q_0} \right]^{1/2}$$

and prices for both years would have essentially the same influence on the measured volume movement even though the volume of transactions in year 0 is far less than what it is in year 1. Given a negative correlation between prices and quantities, this would imply much higher measured rates of growth using the Fisher measure than using the Edgeworth-Marshall measure. This is illustrated with an example in section 8.

Note also that another index number formula often associated with the Edgeworth-Marshall formula, the Bowley-Sidgwick formula, does not satisfy the transactions equality principle either. It is the arithmetic equivalent of the Fisher formula, defined as the arithmetic mean of the Laspeyres and Paasche indexes:

$$Q_{1/0}^{BS} = \left[\frac{\sum p_0 q_1}{\sum p_0 q_0} + \frac{\sum p_1 q_1}{\sum p_1 q_0} \right] / 2$$

Again, it can be seen that even if the volume of transactions is much larger in period 1 than in period 0, with this kind of a case, prices of period 0 and period 1 have about the same impact on the index. (In fact, the Bowley-Sidgwick formula would give an even worse result in this respect than the Fisher formula. Since the geometric mean is always less than the arithmetic mean of two estimates, the higher estimate based on the period 0 prices would have a greater influence on the Bowley-Sidgwick estimate than on the Fisher one.)

Moreover, like the Laspeyres formula, the Bowley-Sidgwick formula is also biased. It does not pass the time reversal test, so that we have:

$$Q_{0/1}^{BS} = \left[\frac{\sum p_1 q_0}{\sum p_1 q_1} + \frac{\sum p_0 q_0}{\sum p_0 q_1} \right] / 2 \neq 1 / Q_{1/0}^{BS} = 2 / \left[\frac{\sum p_0 q_1}{\sum p_0 q_0} + \frac{\sum p_1 q_1}{\sum p_1 q_0} \right]$$

and so, like the Laspeyres formula, although much less so, it is subject to chain index drift.

The SNA93 manual recommends (see 16.76) that a set of fixed-price estimates be calculated in addition to the chain Fisher estimates, which would be rebased and (backward) linked about every five years. This is an excellent recommendation, which recognizes the limitations of annual-linked chain measures for certain kinds of analysis.

The SNA93 manual says nothing on where the base year should be in the finalized estimates for a five-year span in such a chain volume measure, although a seminal study by Szulc (1998) indicated that this is a matter of considerable importance. Comparisons using Canadian data on gross domestic expenditures showed that price indexes calculated according to an annual-link chain Fisher formula were poorly approximated by chain Laspeyres indexes with five-year or ten-year links, but were quite well approximated by chain fixed-basket indexes with the same linking frequency if these employed a mid-year basket. It is reasonable to assume that a similar relationship would hold for volume comparisons, favouring the choice of mid-period base prices for constant-price volume aggregates.

In fact, such was the practice of the Office of National Statistics in the United Kingdom in compiling their volume estimates until they moved to an annual-link chain Laspeyres format.[11] Expenditures for 1983 to 1987 were at 1985 prices, expenditures for 1988 to 1992 at 1990 prices and so on. Because a given year was never more than two years removed from the base year, the base year prices were more likely to be representative of that year, than they would have been if the base year were the initial year of a five-year span, and the given year were the final year or penultimate year of the span.

In their analysis of the differences between annual-linked and published estimates for both expenditure categories and industry estimates Tuke and Brown (2003) note that from 1995 to 2001 these are never larger than 0.2% in absolute terms. These remarkably small differences provide a good indication of the extent to which the choice of a central base year secures a lot of the benefits that can be derived from annual linking. It is likely that the differences would have

[11] See Lynch (2003).

been considerably greater had the Office of National Statistics followed SNA68 conventions and made the base year of their volume estimates the initial year of each span.

Given that the official chain measures should be Edgeworth-Marshall measures, the fixed-price volume measures should not be Laspeyres aggregates, but should be extensions of the series used to calculate the Edgeworth-Marshall links, i.e. for the most recent period they would have the formula:

(4-10) $\sum p_{01} q_y ; y = -2, -1, 2, 0, 1, 2 ...$

These estimates would have to be calculated up to year 6, and possibly to year 10, before they were replaced by new volume series with base prices of years 5 and 6. For the five preceding years, the chained volume estimates would have the formula:

(4-11) $f \times \sum p_{-5-4} q_y = (\sum p_{01} q_{-2} / \sum p_{-5=4} q_{-2}) \times \sum p_{-5-4} q_y ; y = -7, -6, -5, -4, -3$

where the series are at constant prices of the years -5 and -4. Extension to earlier five year spans is obvious. Note that with these links the annual movement for the year -2 is based on base prices for the years -5 and -4, and not those of years 0 and 1.

Generally speaking, calculating fixed-price volume aggregates using base prices over two years rather than a single year would tend to create smoother volume estimates with reduced amplitude, any anomalies in a single year base being smoothed out in a two-year base period. So in two respects, the centrality of the base period, and the fact that a double year base is used, these fixed-price aggregates would be superior to the old SNA68 Laspeyres volume aggregates.

Although the ultimate length of any fixed-price span would only be five years, due to the inevitable lags in rebasing the length of the current span would be much longer. For example, in Canada the monthly GDP estimates could probably only have been switched to 1997-8 prices with the July 2001 update, replacing 1992-93 prices that would have been taken back to January 1992. So the series at 1992-93 prices would have been calculated for January 1990 to June 2001, a period of 11 years and six months. (If the fixed-price series were a Laspeyres series with a 1992 base, it would be a year shorter, ending in June 2000.)

5. Subannual Values of Annually Linked Chain Aggregates

Most countries publish GDP estimates at constant prices both quarterly or monthly, and these subannual estimates are of at least as great interest to users as the annual estimates. Unfortunately, papers on chain volume aggregates written by interested economists or even by official statisticians have tended to ignore the difficulties in creating quarterly series with annual links. Most papers on the subject deal with the characteristics of different formulae -- Fisher, Montgomery-Vartia and so forth -- for individual (read annual) links and one is left with the impression that these same properties of the annual links also exist for the quarterly growth ratios. As a general rule, this is not so, and some formulae which look good in terms of annual links nevertheless look much worse if one considers what kind of quarterly estimates they will generate.

One of the few works that does recognize this problem is chapter XVI of SNA93. Unfortunately, its conclusion, that "chain indices should only be used to measure year-to-year movements and not quarter to quarter movements" is unduly defeatist. This conclusion seems to be based on its observation "that if it is desired to measure the change in prices or volumes between a given month, or quarter, and the same month, or quarter, in the following year, the change should be measured directly and not through a chain index linking the data over all the intervening months, or quarters" (SNA93, section 16.49).

However, annually linked volume aggregates whose constant price structure changes every year are not much different from the old SNA68 volume measures. The growth rates measured between consecutive quarters would all be comparable (i.e., they would satisfy the proportionality test) except for the first quarter. Over a 15-year span covering 59 quarterly changes, the SNA68 aggregates would provide 57 comparable quarterly changes, which is 95% of the total (the first quarter changes for the years following link years being non-comparable). Chain volume aggregates would provide 45 comparable quarterly changes, or 75% of the total. For monthly GDP series, only January monthly changes would be non-comparable, so comparable monthly changes would be 91.7% of the total.

And for that matter, if Edgeworth-Marshall volume aggregates were calculated, the only distortion in a fourth quarter to first quarter comparison would be in the replacement of one set of base prices by another, with both sets being very current and sharing half of their prices in common. In many cases the impact of the change in price structure would be negligible. This would also be true of four-quarter comparisons, which would have to contend with a single link at the year and not on four quarterly links as SNA93 postulates.

Users of SNA data want to have one official set of quarterly and annual estimates, and not go to one series for annual growth rates and another, possibly with quite different annual movements, for quarterly growth rates. Surely users should be provided with overlapping sets of direct volume aggregates that provide meaningful quarterly growth rates where the chain volume aggregates do not do so. In many cases, these alternative series would likely only provide assurance that the quarterly growth rates generated by the chain volume aggregates are not very different from those provided by measures of pure volume change. As was noted in the previous section fixed-price volume aggregates with base prices defined over two years even if finally defined for a five-year span could be initially calculated over a span of 10 to 11 years, so there would be a lot of overlap between fixed-price volume aggregates, ensuring some kind of a fixed-price measure for all quarterly and four-quarter comparisons.

The quarterly volume index estimates equivalent to the Fisher annual estimates would have the following formula:

$$\text{(5-1)} \quad Q^{F}_{1q/0} = \left[Q^{L}_{1q/0} \times Q^{P}_{1q/0}\right]^{(1/2)} = \left[\frac{\sum p_0 q_{1q}}{\sum p_0 q_0} \frac{\sum p_{1q} q_{1q}}{\sum p_{1q} q_0}\right]^{(1/2)}.$$

This is the same formula as (4-9) except that all period 1 quantities and the Paasche index's period 1 prices are now for a specific quarter rather than for a year. As far as this particular index comparison goes, proportionality is still satisfied, since what we have is still the geometric mean of two fixed-price volume indexes. Factor reversal is also satisfied since:

$$Q^F_{1q/0} \times P^F_{1q/0} = \left[\frac{\sum p_0 q_{1q}}{\sum p_0 q_0}\frac{\sum p_{1q} q_{1q}}{\sum p_{1q} q_0}\right]^{(1/2)}\left[\frac{\sum p_{1q} q_0}{\sum p_0 q_0}\frac{\sum p_{1q} q_{1q}}{\sum p_0 q_{1q}}\right]^{(1/2)} = \frac{\sum p_{1q} q_{1q}}{\sum p_0 q_0}$$

However, since there is also annual rather than quarterly linking, the quarterly growth ratios of this index do not satisfy proportionality. What we have is

$$Q^F_{1q/0} / Q^F_{1q-1/0} = \left[\frac{\sum p_0 q_{1q}}{\sum p_0 q_0}\frac{\sum p_{1q} q_{1q}}{\sum p_{1q} q_0}\right]^{(1/2)} / \left[\frac{\sum p_0 q_{1q-1}}{\sum p_0 q_0}\frac{\sum p_{1q-1} q_{1q-1}}{\sum p_{1q-1} q_0}\right]^{(1/2)}$$

$$= \left[\frac{\sum p_0 q_{1q}}{\sum p_0 q_{1q-1}}\frac{\sum p_{1q} q_{1q}}{\sum p_{1q} q_0} / \frac{\sum p_{1q-1} q_{1q-1}}{\sum p_{1q-1} q_0}\right]^{(1/2)},$$

and even in the absence of quarterly price change by any commodity, the growth rate will generally be different from one due to price movements between the base year and the quarters of the current year. The problem stems from the Paasche component of the Fisher index, and not from the Laspeyres component; a direct Paasche index, because its relative price structure changes every period, does not satisfy the proportionality axiom for multi-period comparisons; therefore, neither does the direct Fisher index.

Formula (5-1) can be simplified by replacing its Paasche component with a fixed-price index reflecting the price structure of year 1:

$$\text{(5-2)} \quad Q^{GMFP}_{1q/0} = \left[Q^L_{1q/0} \times Q^1_{1q/0}\right]^{(1/2)} = \left[Q^0_{1q/0} \times Q^1_{1q/0}\right]^{(1/2)} = \left[\frac{\sum p_0 q_{1q}}{\sum p_0 q_0}\frac{\sum p_1 q_{1q}}{\sum p_1 q_0}\right]^{(1/2)},$$

where the superscript GMFP indicates a geometric mean of fixed-price volume indexes. Following the second equality sign, the superscripts of the volume indexes indicate what year they take their fixed price structure from. Thus, $Q^0_{1q/0}$ is identical to $Q^L_{1q/0}$, the Laspeyres index. In some cases, in the absence of quarterly price data, such a formula might be used to represent a Fisher volume index, although strictly speaking it is not one. A GMFP index does satisfy proportionality for its quarterly growth rates:

$$Q^{GMFP}_{1q/0} / Q^{GMFP}_{1q-1/0} = \left[\frac{\sum p_0 q_q}{\sum p_0 q_0}\frac{\sum p_1 q_{1q}}{\sum p_1 q_0}\right]^{(1/2)} / \left[\frac{\sum p_0 q_{1q-1}}{\sum p_0 q_0}\frac{\sum p_1 q_{1q-1}}{\sum p_1 q_0}\right]^{(1/2)} = \left[\frac{\sum p_0 q_{1q}}{\sum p_0 q_{1q-1}}\frac{\sum p_1 q_{1q}}{\sum p_1 q_{1t-1}}\right]^{(1/2)}$$

as one would expect of the geometric mean of the growth rates of two fixed-price volume indexes. However, the GMFP index does not satisfy the factor reversal test when multiplied by its equivalent geometric mean fixed basket (GMFB) index:

$$Q^{GMFP}_{1q/0} \times P^{GMFB}_{1q/0} = \left[\frac{\sum p_0 q_{1q}}{\sum p_0 q_0}\frac{\sum p_1 q_{1q}}{\sum p_1 q_0}\right]^{(1/2)}\left[\frac{\sum p_{1q} q_0}{\sum p_0 q_0}\frac{\sum p_{1q} q_1}{\sum p_0 q_1}\right]^{(1/2)} \neq \frac{\sum p_{1q} q_{1q}}{\sum p_0 q_0}$$

These geometric mean indexes are the quarterly indexes that are closest to Fisher indexes while retaining the proportionality property. Rather than leaving it to the different national statistical agencies to decide what to do, this is what SNA 1993 should have prescribed for quarterly measures when it advised the adoption of the Fisher formula for annual measures. However, so far these measures have not been adopted by either the United States or Canada, the only countries, so far, that have adopted the Fisher formula for their chain volume measures.

Often there is a greater level of detail available for both price indexes and value series at the annual level than at the quarterly level, so there is a need to adjust a quarterly volume indicator to annual benchmarks. For a geometric mean volume index, it would be inappropriate to directly adjust it so that it averaged to a Fisher volume index. Instead, one would want to benchmark the quarterly chain Laspeyres volume index to its annual benchmarks, and the quarterly chain fixed-basket index compatible with a chain Paasche volume index to its annual chain Paasche benchmarks, and then calculate the geometric mean of the adjusted series.

Since the additivity problem is usually discussed in a general sense, many people may be unaware of the crucial distinction between additivity of quarters to years and other types of additivity (over commodities, industries or regions) in calculating annually-linked chain volume indexes. Generally speaking, chaining destroys all types of additivity except the addditivity of quarterly values to annuals, where that additivity exists to begin with. Therefore it is simply not true that there is not much difference, as far as additivity is concerned, between chain fixed-price volume measures such as the Edgeworth-Marshall series and other measures such as Fisher or Sato-Vartia that do not have this additivity property.

Unfortunately, the implementation of chain Fisher volume measures by both the U.S. Bureau of Economic Analysis (BEA) in the U.S. National Accounts and by Statistics Canada in the CSNA has muddied these distinctions. Both have imposed additivity on their quarterly chain Fisher measures, although it is illogical to do so. The BEA calculates quarterly-linked chain Fisher indexes and benchmarks them to annually-linked chain Fisher benchmarks. The CSNA has also calculated quarterly-linked chain Fisher indexes, but has not benchmarked these estimates to anything at all; the annual estimate is the arithmetic mean of the quarterly estimates.

Formula (5-1) is a better way to decompose an annually-linked index than using a quarterly-linked distributor, if deflation occurs at the same level of detail for the annual and the quarterly levels. If this is not the case, the appropriate thing to do would be to adjust a quarterly distributor of chain Laspeyres annual benchmarks and another distributor to chain Paasche annual benchmarks, and then month by month calculate the geometric mean of the two series. (Assuming that it were appropriate to have a quarterly-linked chain Fisher distributor, it should be adjusted to the chain Fisher benchmarks by using the chain Laspeyres quarterly series as the distributor for the chain Laspeyres benchmarks and the chain Paasche quarterly series for the chain Paasche benchmarks.) Whatever distributors were used, the quarterly index numbers would not average to the annual benchmarks.

As for the CSNA measure, given that the quarterly index numbers are based on quite different price structures, their arithmetic average is essentially meaningless. It would have been more appropriate to calculate their annuals as a geometric mean since the chain series is nothing but a strand of quarterly growth ratios linked together.

Both the BEA and Statistics Canada seem to have had misgivings about the consequences that adopting the Fisher formula would have for the additivity of their estimates, and this has kept both of them from calculating annuals and quarterlies in a way that is logical and consistent. This is especially surprising for the BEA because they have been publicly dismissive of the value of additivity in national accounting (see, for example, the 2000 paper by Ehemann et al.).

Why doesn't the BEA practice what it preaches? Or, to turn the question around, why doesn't it preach what it practices? Given that it is so unwilling to give up additivity of quarterly estimates to annuals that it will impose this property, against all logic, on its chain Fisher estimates, why doesn't the BEA advocate and implement chain fixed-price volume aggregates like the Edgeworth-Marshall measures that would legitimize this additivity? And the same goes for Statistics Canada.

The use of quarterly-linked chain estimates by both the United States and Canada has a serious potential for degrading the quality of growth estimates, so it merits further comment. As Hill remarks in SNA93:

"[A] chain index should be used when the relative prices in the first and last periods are very different from each other and chaining involves linking through intervening periods in which the relative prices and quantities are intermediate between those in the first and last periods" (SNA93, p.388).

If, for example, one wants to know the difference in output between the first quarter of 2001 and the first quarter of 2003, linking eight times through different quarters will hardly give us in each and every case a relative price structure that departs smoothly from that of the initial quarter in the direction of the terminal relative prices in the next quarter and that will inevitably be intermediate between those of the first and last quarters. Suppose, for example, one were measuring gross output of the air transport sector. Does it really make sense to link through the prices of third quarter and fourth quarter 2001, given the highly abnormal pricing situation in the wake of the September 11th terrorist atrocities? The consequences of linking where one shouldn't be are reduced because the Fisher formula has the time reversal property, but this in no way justifies linking inappropriately.

Some commodities are seasonally disappearing and this is especially a problem for northern countries like Canada. Prices may well be missing for one or both quarters of a quarterly comparison. More seriously if quantities go to zero, one must calculate output relatives with a zero base, which makes the index number undefined.[12]

This problem is only partially resolved by the seasonal adjustment of economic time series. First, even if a statistical agency only publishes GDP at constant prices as a seasonally adjusted series, it is sound statistical practice to calculate the same aggregate unadjusted for seasonal variation. Any chain linking procedure that makes the calculation of such raw estimates impossible, or requires so much tinkering to calculate them that their comparability with the official seasonal adjusted series is compromised, has little to recommend it.

[12] In principle, for a formula like the Fisher, if the volume relative of one of its components is undefined due to zero production in the base quarter, the calculation can be redefined as an aggregative formula in which the production value for that commodity simply disappears from the sum for the base quarter. This is the advantage of an aggregative formula like Fisher (or Edgeworth-Marshall) over a log-change formula like Törnqvist, since there is no way the latter can be defined, except as a function of volume relatives. However, in a production context, it might be inconvenient to calculate the estimates using an aggregative formula rather than a sum of weighted averages formula.

Second, one is necessarily resorting to highly artificial base prices. Certainly seasonal adjustment procedures can arrive at a first quarter price for field-grown corn one way or another, but it is not and can never be a solid number, unlike its annual unit price.

Although both BEA and Statistics Canada procedures involve quarterly linking, the BEA procedures are better, because at least the quarterly-linked estimates are adjusted to good annual benchmarks. The Canadian estimates are not, which leaves open the possibility of substantial chain index drift in one direction or another.

Another danger with the CSNA procedure is that more than anything else, the quality of output estimates at constant prices depends on the level of disaggregation at which the estimation takes place.[13] More disaggregated information on prices, revenues or production volumes is, and always will be available at the annual than at the quarterly level. Creating a methodology that requires quarterly estimates encourages the calculation of production estimates to occur at too gross a level of detail.

One of the properties of the Fisher formula is proportionality; i.e., if all quantities of all commodities increase by the same percentage, that will also be the rate of increase of the Fisher volume index:

$$\left[\frac{\sum p_0 \lambda q_1}{\sum p_0 q_0} \frac{\sum p_1 \lambda q_1}{\sum p_1 q_0}\right]^{(1/2)} = \left[\frac{\lambda \sum p_0 q_1}{\sum p_0 q_0} \frac{\lambda \sum p_1 q_1}{\sum p_1 q_0}\right]^{(1/2)} = \lambda \left[\frac{\sum p_0 q_1}{\sum p_0 q_0} \frac{\sum p_1 q_1}{\sum p_1 q_0}\right]^{(1/2)}$$

Note that the geometric mean index shown in (5-1) also has the proportionality property for quarterly changes. On the other hand, the BEA measures only approximately have this property for quarterly changes. The quarterly changes of their quarterly-linked series would have this property prior to benchmark adjustment, but not afterwards. In other words, the quarterly data may show no quantity change from one quarter to another in any commodity, but due to benchmark adjustment an increase or decrease in output will nonetheless be indicated. And this could happen even if the quarterly distributor and the annual benchmarks were based on the same data for the same commodities. The BEA documents on their chain measures have not paid any attention to this important caveat.

Nonetheless, the BEA measures do satisfy proportionality for annual changes. This is not true of the Statistics Canada measures. Calculated as the average of the quarterly Fisher estimates, different years are not compared based on the same sets of relative prices, so the annual estimates could show an increase or a decrease even if all outputs were unchanged. In other words, the BEA has chosen to give priority to their annual growth rates; Statistics Canada to their quarterly growth rates. Obviously, the BEA has made the better choice. The quality of quarterly estimates will never be up to the standard of the annual estimates.

However else the Statistics Canada changes its methodology for calculating chain volume estimates for the CSNA, the agency should switch to chaining annually regardless of the choice of formula. Chaining quarterly with no annual benchmarks is contrary to SNA93, contrary to BEA practice, contrary to Eurostat practice, and contrary to common sense.

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[13] On this topic, see the paper by Horner (1971).

Quarterly- or monthly-linked indexes do have their uses. For example, they are a required adjunct if one is calculating seasonal-basket price indexes and wishes to make some sense of quarterly or monthly price movements. However, they are only useful as adjuncts to official series, rather than *as* official series.

Besides the Fisher formula, a number of log change formulae have been recommended for measuring volume indexes, including the Törnqvist, Montgomery-Vartia, and Vartia-Sato ones. All conform to the following general formula:

$$Q_{1/0}^{LC} = \prod \left(q_1 / q_0\right)^{w} = \sum w(\ln(q_1) - \ln(q_0)),$$

whence the name log-change indexes.

The Törnqvist index is mentioned as an alternative to the Fisher index in SNA93. The Montgomery-Vartia formula was recommended as the best technical formula by consultants to Eurostat for its good properties, despite the fact that it does not meet the proportionality test. By the way, this report dismissed the Fisher formula from consideration because it is not even weakly consistent in aggregation.[14] The closely related Sato-Vartia formula has also attracted considerable interest. Like the Fisher formula, it passes both the factor reversal and the time reversal test, and it is exact with respect to a CES aggregator function.[15]

However, the Törnqvist formula has a serious problem when production goes to zero for a commodity. Technically the index becomes undefined, since the logarithm of zero does not exist. This is a real problem for calculating quarterly and especially monthly GDP. Production of some commodities does go to zero for seasonal and other reasons. Vartia states in his paper that the Sato-Vartia formula will properly handle quantities or prices going to zero. However, this is only true if the index weights and component index numbers are changing at the same time. If one is linking annually but calculating quarterly or monthly index estimates, and output does not go to zero in all quarters of the year, the Sato-Vartia index will also be undefined. For calculating subannual series then, it is not more robust than the Törnqvist formula, since it is much more likely that production will drop to zero for a few months than for an entire year.

So the Sato-Vartia index is exact for the CES aggregator function yet becomes undefined for situations that frequently occur in calculating quarterly economic accounts. This is why one should put quotes around the word "superlative" when applied to an index that is exact for an aggregator function; it may be anything but superlative in terms of its index properties.

6. Analysis of Changes for Quarterly Chain Volume Aggregates

Any annually-linked chain volume aggregate will have a problem of non-comparability for quarterly changes for the first quarter due to the switch from one relative price structure to another. However, it is possible to precisely measure the distortion due to this switch. The quarterly percent change in the volume aggregate if there were no change in the relative price structure would be:

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[14] See Al et al. (1986, p.354) for dismissal of the Fisher formula and p.355-56 in this same reference for their endorsement of the Montgomery-Vartia formula, which they call the Vartia-I formula.

[15] See the articles by Vartia (1976) and by Reinsdorf and Dorfman (1999) for more on this fascinating formula.

(6-1) $100 \times \sum \bar{p}_{t-2t-1}(q_{t,1} - q_{t-1,4}) / \sum \bar{p}_{t-2t-1} q_{t-1,4}$.

The difference between this quarterly percent change and the actual change in the official index is the measure of the interaction between the change in the relative price structure and output change. In most cases, this difference would not be great. Annual prices are less volatile than quarterly prices to begin with, and a two-year moving average of annual prices would help iron out the fluctuations in annual movements. However, where the difference was considerable for an official aggregate, a publication release could draw attention to the pure volume change measure, and deemphasize the official quarterly estimate of volume change.

The same principle would apply in the case of four-quarter percent changes. The percent change in the chain volume aggregate from the fourth quarter of year t-1 to the fourth quarter of year t, if there were no change in the relative price structure, would be:

(6-2) $100 \times \sum \bar{p}_{t-2t-1}(q_{t,4} - q_{t-1,4}) / \sum \bar{p}_{t-2t-1} q_{t-1,4}$,

and the difference between this percent change and the official estimate would show the distortion of the measured growth rates due to annual linking.

It should be noted that if the Edgeworth-Marshall aggregates were linked at the fourth quarter and not at the year, then the four-quarter percent change of the official estimates would be a measure of pure volume change:

(6-3) $100 \times \sum \bar{p}_{t-1t}(q_{t,4} - q_{t-1,4}) / \sum \bar{p}_{t-1t} q_{t-1,4}$,

and there would be no distortion due to changes in relative price structure. However in this case the annual price movements would be compromised.

In Statistics Canada, linking at the fourth quarter is the practice for some, but not all, price indexes whose baskets are updated once a year.[16] Regardless of whether this is the best practice for those price indexes, it should be observed that the context is different here. There are no annual benchmarks for the price indexes that are linked at the fourth quarter. Their annual data are derived from their quarterly data. For SNA volume aggregates, there are annual benchmark estimates for many series and in some cases sub-annual estimates are non-existent. Hence, linking at the fourth quarter would be inadvisable; it would be better to link at the year.

A couple of recent papers by Rossiter (2000) and Whelan (2000), American economists who work outside of the BEA have outlined quite arcane calculations to derive the contributions of components to growth for the US chained volume measures.[17] Neither seems to give sufficient attention to the fact that when comparing chained volume estimates across link periods, strictly speaking, no precise calculation of contribution shares to growth is possible. Rossiter does acknowledge a residual component, reflecting the difference between the sum of component contributions and the actual change in GDP. Over the 1991Q1 to 1998Q1 this residual, reflecting interaction between volume change and shifts in price structure accounts for 7.6% of the

[16] The non-residential building construction price index and the apartment building construction price index are linked at the fourth quarter; the air fares index is linked at the year.

[17] The Rossiter paper also shows contributions to change summing to 100, rather than to the percent change of total GDP over the period, a formulation that can break down completely if overall GDP growth is zero or nearly so.

measured growth in GDP. It seems to be an abuse of language to call such estimates contributions to change, when they are at best partial contributions to change.

For longer time periods, say the segment of 1991Q1 to 1998Q1 that Rossiter considers in his paper, output comparisons based on a single set of constant prices, while problematic, are possible. Indeed, it would seem from the discussion in section 4 that even a quarterly Laspeyres series at 1992 prices (1992 being a base year for the BEA and the CSNA both) would likely be calculated from 1990Q1 to 2000Q2, more than covering the segment at issue. Component contributions to the percent change in the aggregate could be simply calculated and would be true contributions, not having a residual component. Their principal drawback is that they relate to an analytical series and not to the official chain measure of volume change.

Such fixed-price series would also be useful for the dating of business cycles. This is a delicate task at best, and it would become largely impossible if one had only chained volume estimates to work with. The fixed-price series would be useful to monitor the changes in volume shares of components of GDP from one period to another where chain volume estimates are non-additive. (As discussed above, for the most recent year or years, the Edgeworth-Marshall estimates would be additive, so one could compare, say, the share of ICT products in current quarter GDP with the share in the previous quarter cleanly and correctly.) In his paper, Whelan acknowledges that chain measures are not particularly useful for this task and argues instead that one should base comparisons of component shares on expenditures at current prices, ignoring the inflationary distortions inevitable in such a procedure. This is simply ridiculous, and would mean a big step backward for analysis of economic statistics if it ever became commonplace.

Yet BEA economists Ehemann et al (2000) agree with Whelan and argue that shares of volume series are inherently meaningless because investment in computer equipment exceeds investment in software with a 1996 base but not with a 1992 base. I fail to see the point of this argument. Not all bases are created equal, which is precisely why there has been a move to annual chain measures. Anyone who wanted to know if software’s share in the volume of investment between 1996 and 1999 was growing would prefer estimates at 1996 prices; estimates at 1992 prices would be rejected given a choice. The two base years are not on an equal footing. But estimates at one set of prices or the other would inevitably be preferred to comparing investment shares between the two years each at their own set of prices. What is to be gained by such an apples and oranges comparison?

7. An Aside on Price Indexes for GDP

The implicit price indexes for GDP that are derived from the Edgeworth-Marshall volume aggregates would not be the most suitable price measures for the components of GDP. In order to be consistent with the calculation of the volume measures, it would make sense to also calculate annually-linked chain Edgeworth-Marshall price indexes. The index would be calculated as:

(7-1) $$P_{y/0}^{EM(Ch)} = \prod_{t=1}^{y} P_{t/t-1}^{EM}$$

where

$$P_{1/0}^{EM} = (\sum p_1(q_0 + q_1)/2)/\sum p_0(q_0 + q_1)/2 = (\sum p_1(q_0 + q_1))/\sum p_0(q_0 + q_1)$$

This may be contrasted with the similar formula for a Walsh chain price index:

(7-2) $$P_{y/0}^{W(Ch)} = \prod_{t=1}^{y} P_{t/t-1}^{W}$$

where $P_{1/0}^{W} = (\sum p_1\sqrt{q_0 q_1} / \sum p_0\sqrt{q_0 q_1}$,

and where the prices are weighted by the geometric mean of the quantities in years 0 and 1.

Note that the calculation of average quantities over the two years for the Edgeworth-Marshall price index is based on a simple arithmetic mean whereas the calculation of average prices over the two years is based on a weighted harmonic mean. There is no inconsistency here. As can be seen, there is no difference in terms of result between weighting by the mean of quantities or the sum of quantities. The most natural way of combining the two periods' quantities is to add them together, just as the most natural way of calculating their average price for a homogeneous product is to take the unit value. It is not more natural to take the geometric mean of quantities in years 0 and 1-- that is, it is not more natural to calculate a Walsh price index -- than to take their sum. Moreover, the Walsh index will ignore any commodity that is available in only one of the two periods since the geometric mean of a positive value and a zero value is zero. (On the other hand, this does mean that a commodity will simply fall out of the index basket for a Walsh price index where an Edgeworth-Marshall index must impute for a missing price.)

In some cases, for commodities with highly volatile production profiles, a two-year basket may not be adequate to generate a stable weighting pattern.[18] For such commodity groups or industries, the chain Edgeworth-Marshall series could be replaced with similar indexes based on three- to five-year baskets. At higher levels of aggregation, the chain Edgeworth-Marshall formula could continue to be employed.

Such an adaptation of the Edgeworth-Marshall formula would not pass the time reversal test as defined above. However, that is not a big problem; it would still break the association between basket change and price change that makes the use of the Laspeyres formula a hazard in chain computations. Unlike the chain Laspeyres series, an Edgeworth-Marshall series would not be much subject to chain index drift.

Also, the Edgeworth-Marshall formula would have to be adapted to accommodate monthly baskets for seasonal commodity groups, using the Rothwell (1958) formula or the Balk (1980a) formula or both. These formulae will not be shown or discussed here beyond saying that there would be more reason to use the Balk formula the more irregular the seasonal pattern of the commodity group in question, since the Rothwell formula postulates a constant seasonal pattern. Also, while the long revision period required to properly calculate a Balk price index has kept it

[18] Within Statistics Canada, the new housing price index, non-residential building construction price index and apartment building construction price index all have three-year baskets; the farm product price index has a five-year basket.

from being employed in consumer price indexes, and for the most part in industry price indexes, the much greater tolerance of national accounts estimates for revisions should not keep it from being used in price indexes for national accounting aggregates.

Although there is a dichotomy between price and volume indexes with the same formulas being used for both, there is no reason to believe that the best price index formula for a commodity group is also necessarily the best volume index formula, and still less reason to believe that this formula will satisfy the strong factor reversal property.

Prices have their cyclical and irregular movements as do production flows, and this is one reason for favouring fixed-price series with a two-year base period, but it is less likely that a volume series will require a three- to five-year base period than it is that a price index will require a three- or five-year basket. As for seasonal commodities, the Rothwell and Balk formulas are strictly price index formulas; no-one has ever used them to measure the volume change for seasonal commodities and no-one ever will. There is really no place for special treatment of seasonal commodities in volume indexes, except in the special case where a good can be usefully considered to be a different commodity in every month or quarter of the year. Although such cases do arise, they are rare, and at the level of aggregation at which national accountants operate, probably non-existent.

Therefore, it is really quite immaterial that the Fisher formula satisfies the strong factor reversal test and the Edgeworth-Marshall doesn't, since if one wanted to calculate the best price index for national accounting aggregates, one would necessarily be adjusting the Fisher price index to handle cyclical or seasonal commodities in a special way, and factor reversal would no longer apply.

These chain Edgeworth-Marshall price series should have their fixed-basket counterpart in a price index defined as:

$$P_{y/0}^{EM} = \frac{\sum p_y(q_0 + q_1)}{\sum p_0(q_0 + q_1)}; y = -2,-1,0,1,2,...$$

For the five years previous it would be chained as follows:

$$\begin{aligned} P_{y/0}^{EM} &= f \times \frac{\sum p_y(q_{-5} + q_{-4})}{\sum p_{-5}(q_{-5} + q_{-4})} \\ &= \left(\frac{\sum p_{-2}(q_0 + q_1)}{\sum p_0(q_0 + q_1)} \Big/ \frac{\sum p_{-2}(q_{-5} + q_{-4})}{\sum p_{-5}(q_{-5} + q_{-4})} \right) \times \frac{\sum p_y(q_{-5} + q_{-4})}{\sum p_{-5}(q_{-5} + q_{-4})} \\ &= \frac{\sum p_{-2}(q_0 + q_1)}{\sum p_0(q_0 + q_1)} \times \frac{\sum p_y(q_{-5} + q_{-4})}{\sum p_{-2}(q_{-5} + q_{-4})}, \qquad \text{for } y = -7,-6,-5,-4,-3. \end{aligned}$$

Earlier spans would be linked along the same lines. Arguably, if the price index were quarterly or monthly in frequency, the chain price index might be linked at the terminal quarter or terminal month, rather than the terminal year.

These would not be chain Edgeworth-Marshall price indexes; they would be chain Lowe indexes, but they would be based on baskets used in the chain Edgeworth-Marshall price indexes, and they would benefit from the use of two-year baskets, which would tend to give them a smaller amplitude than their chain Laspeyres counterparts.

8. Examples of Volume Measures for Declining and Rapidly Expanding Industries

To get a feel for the importance of the transactions equality principle, it is useful to illustrate this with a pair of examples. Table 1 lists data for a declining industry with four goods. Output and prices are decreasing for all of the goods in this industry, and there is the expected negative correlation between price changes and output. As can be seen, the volume of production in year 0 is only about five eighths its level in base year 0, evaluated at base year prices. Particularly significant is the decline in output of good 3, accompanied by a doubling in its price.

Table 1. Hypothetical Prices and Quantities for a Declining Industry

	p_0	q_0	p_0q_0	p_1	q_1	$Q_{1/0}$	p_0q_1
Good 1	1.0	3,000	3,000	0.5	1,940	0.647	1,940
Good 2	1.0	2,000	2,000	0.4	1,600	0.800	1,600
Good 3	1.0	1,000	1,000	2.0	180	0.180	180
Good 4	1.0	1,000	1,000	0.5	650	0.650	650
All goods			7,000				4,370

Source: Ehemann et al. (2000, p.6).

Table 2 shows the relative shares of the four commodities for Laspeyres, Paasche, Edgeworth-Marshall and Fisher volume indexes.[19] Note particularly the much stronger relative share of good 3 for the Paasche index, a consequence of good 3 doubling in price. While good 3 (along with good 4) is the least important commodity for the Laspeyres index, it is the dominant commodity for its Paasche counterpart. The Fisher shares are a weighted average of the Laspeyres and Paasche shares, but they differ very little from a simple average of these shares. For good three, for example, there is a 28.9% share, which is slightly larger than the 28.0% share if one takes the mean of the Laspeyres and Paasche shares. In contrast, because the Edgeworth-Marshall shares are based on average prices for years 0 and 1 that are calculated as unit values, they give more importance to year 0 than year 1, and are much closer to the Laspeyres shares than the Paasche shares. The share for good 3 in particular is much lower, at 19.8%, which is only about three eighths more than the Laspeyres share.

[19] The relative shares for the Fisher index were calculated using the formula shown in Balk (2004; p.109) with the appropriate substitution of price indexes for volume indexes and vice versa.

Table 2. Relative Importance of Goods for Different Volume Indexes for the Declining Industry Example

	Laspeyres	Paasche	Edgeworth-Marshall	Fisher
Good 1	0.429	0.313	0.413	0.367
Good 2	0.286	0.167	0.251	0.222
Good 3	0.143	0.417	0.198	0.289
Good 4	0.143	0.104	0.138	0.122

Table 3 shows the volume indexes generated in terms of this example. Given the negative correlation between prices and quantities, the Paasche index is lowest and the Laspeyres index is the highest. Note that the Fisher index is only slightly less than the Bowley-Sidgwick one (as it must be, since the geometric mean of any two numbers will always be less than the arithmetic mean), but the Edgeworth-Marshall index is much greater than either, being considerably closer to the Laspeyres measure than the Paasche one. The Walsh index is also larger than either of the crosses of Laspeyres and Paasche indexes, but less than the Edgeworth-Marshall index. In relative terms, lying closer to the Fisher index than to the Edgeworth-Marshall one.

The Edgeworth-Marshall measure gives a fairer measure of the decline in output in year 1 for this declining industry. The Fisher measure gives excessive importance to the price structure of year 1, since that production is much lower by any measure for this declining industry in the second year. In particular, it gives too high a share to good 3, whose production in year 1 had essentially collapsed.

Table 3. Comparison of Volume Indexes for the Declining Industry Example

Paasche	0.478
Fisher	0.546
Bowley-Sidgwick	0.551
Walsh	0.562
Edgeworth-Marshall	0.593
Laspeyres	0.624

If one simply inverts the previous situation, so that the prices and quantities of period 0 are those of period 1 and vice-versa, one gets the case of an expanding industry. Now there is expansion for all four goods in the industry, but particularly for good 3 whose output more than quintuples going from year 0 to year 1. Again there is the expected negative correlation between price change and volume change: the good with the largest price increase, good 2, also has the smallest increase in output. Notice that at prices of base year 0, the output of the industry in year 1 is more than double what it was in the previous year 0.

Table 4. Hypothetical Prices and Quantities for a New Expanding Industry

	p_0	q_0	p_0q_0	p_1	q_1	$Q_{1/0}$	p_0q_1
Good 1	0.5	1,940	970	1.0	3,000	1.546	1,500
Good 2	0.4	1,600	640	1.0	2,000	1.250	800
Good 3	2.0	180	360	1.0	1,000	5.556	2,000
Good 4	0.5	650	325	1.0	1,000	1.538	500
			2,295				4,800

Table 5 shows the shares of the different goods using different index formulae. Again, the most important change is for good 3, which is much less important using the Paasche index due to the halving of its price in year 1. Once more the Fisher shares, although representing a weighted average of the Laspeyres and Paasche shares, differ little from their simple average. But the Edgeworth-Marshall index, given the much greater volume of activity in year 1, has shares that more nearly reflect those of the Paasche measure. Also note that its shares for goods 1 and 4 fall outside the bounds defined by the shares of the Laspeyres and Paasche indexes, being slightly higher than the Paasche shares in both cases. The hybrid expenditures on which these shares are based must always lie between the bounds defined by the actual expenditures that determine the Laspeyres shares and the hybrid expenditures that determine the Paasche shares, but this is not true of the expenditure shares themselves.

Table 5. Relative Importance of Goods for Different Volume Indexes for the New Expanding Industry Example

	Laspeyres	Paasche	Edgeworth-Marshall	Fisher
Good 1	0.423	0.444	0.450	0.434
Good 2	0.279	0.366	0.339	0.325
Good 3	0.157	0.041	0.060	0.095
Good 4	0.142	0.149	0.151	0.145

Table 6 shows the rankings of the volume indexes calculated for the expanding industry case. Again, given the negative correlation between prices and quantities the Laspeyres index shows the highest growth and the Paasche index the lowest. The Fisher index is right between them, showing only a little less growth than the Bowley-Sidgwick index (as it must, given that a geometric mean will always be less than an arithmetic mean). In this case, the Edgeworth-Marshall index is inferior to most of the other indexes, exceeding only the Paasche index. The Walsh index is greater than the Edgeworth-Marshall index but less than the Fisher index; in relative terms, it lies closer to the Fisher index.

Table 6. Comparison of Volume Indexes for the New Expanding Industry Example

Paasche	1.602
Edgeworth-Marshall	1.685
Walsh	1.778
Fisher	1.830
Bowley-Sidgwick	1.847
Laspeyres	2.092

More than the previous example, this one illustrates the dangers of employing the Fisher formula due to its failure to satisfy the property of transactions equality. Although the volume of output in the comparison (i.e., current) period is much more important than in the base period, the Fisher formula essentially treats the two periods on equal terms, and so assigns undue importance to good 3. The small share for good 3 in the Fisher volume index belies its important contribution to change. It alone accounts for 43% of the measured 83% growth for this industry, which is surely excessive given that its production really only took off when its prices were cut in half. In the Edgeworth-Marshall index, good 3 is still the most important contributor to growth, but it only accounts for 27% of aggregate industry growth, barely exceeding good 1 which is responsible for 25% of aggregate growth.

Much has been written in recent years about the new economy, and indeed annual chaining of volume measures was introduced in many countries in hopes of achieving more accurate measurement in the new economy. However, the use of the chain Fisher formula in measuring growth of output for the new economy only reduces rather than eliminates the possibility of upward bias in measured growth rates. For this sector, the replacement of chain Fisher measures by their Edgeworth-Marshall equivalents would likely give lower and more meaningful estimates of growth in output.

Besides the consideration of the transaction equality principle, there is also the consideration that in a rapidly growing industry, new products may be introduced at concessionary prices that are not true market prices. There is also the problem of introducing prices for new goods in price index programs, with prices of new goods often proxied rather than priced for some time after their introduction. (These problems are discussed in more detail in Baldwin et al (1997).) If these factors don't create an obvious upward or downward bias in measuring prices for new goods, they certainly reduce the reliability of such measures. This is another reason why, if the volume of activity in two consecutive years is much greater in the second year reflecting an increasing maturity in the industry, it makes more sense to give greater weight to the price structure of the second year (as the Edgeworth-Marshall formula does) than to treat the two years on an equal footing (as the Fisher formula or the Walsh formula does).

The author received comments on an earlier version of this paper concerning realistic situations in which a Fisher price or volume index would allegedly perform better than its Edgeworth-Marshall counterpart. One was when a small open economy suffered a major devaluation of its currency, precipitating a contraction. In this situation there would be a dramatic increase in the price of imported goods, and a big decline in their purchases. Here the Edgeworth-Marshall price index would allegedly be inferior to the Fisher price index because it would closely approximate a Laspeyres price index, its basket being much more like the basket of the base year than of the subsequent year.

Table 6. Comparison of Price Indexes for the Example of a Declining Industry / Open Economy in a Devaluation

Paasche	0.525
Walsh	0.584
Fisher	0.600
Bowley-Sidgwick	0.605
Edgeworth-Marshall	0.624
Laspeyres	0.686

In a sense the declining industry example shown earlier can be reinterpreted in this way, with good 3, whose prices double in a year, representing imported goods. And using its data to construct price indexes rather than volume indexes it can be seen that the Edgeworth-Marshall index does come closer to the Laspeyres index than do any of the others.(See Table 6 above.) Interestingly, the Walsh index, which in terms of its formula would seem to be so similar to the Edgeworth-Marshall index, actually comes the closest to the Paasche index; both the crosses of Laspeyres and Paasche indexes are closer to the Edgeworth-Marshall index than the Walsh index. In fact, it can be easily shown that if one assumes a doubling of prices is sufficient to choke off imports altogether so that good 3 disappears from the basket in year 1, the Walsh index would be virtually identical with the Paasche index, exceeding it by only 0.5%, since in this case good 3 would disappear from its basket just as it does from the year 1 basket.

Note that the Fisher index is only very slightly lower than the Bowley-Sidgwick index, which is just the mean of the Paasche and Laspeyres price indexes.

So which of these index numbers is more reasonable? At year 0 prices, good 3, the imported good has a 10.4% basket share for years 0 and 1 combined, a 14.3% basket share for year 0 only but only a 4.1% share for year 1. The Fisher and Bowley-Sidgwick indexes are lower than the Edgeworth-Marshall index in large part because they treat the two basket shares as being equally valid, which means that the doubling of prices for the import good does not have nearly the same impact on them that it does on the Edgeworth-Marshall index. But it is surely unreasonable to treat the two baskets as having equal validity when at year 0 prices the volume of expenditures in year 1 is only 60% of those of year 0. Thus the Edgeworth-Marshall index produces the most reasonable result.

Nor is there any reason to believe that the use of the Edgeworth-Marshall formula would seriously overweight imports were the year 1 situation to become the depressing new normal. What is being calculated is not a direct Lowe price index with a year 0 and year 1 basket, but a chain Edgeworth-Marshall price index with annual links. If year 2 saw no change in basket shares from year 1, all index links would show the same price increases, whatever their formulas.

The other situation mentioned was the case of high rates of inflation, which would tend to make the Marshall-Edgeworth volume measures more closely resemble Paasche measures, since the more highly inflated values in year 1 would, other things being equal, have a greater impact on the determination of base prices.

There are two points to make about this. First, the assumption that an Edgeworth-Marshall volume index would necessarily be closer to a Paasche volume index in a high inflationary situation needs to be greatly qualified. It can easily be shown that if one doubles every price in year 2 for the declining industry case shown in Table 1, the Edgeworth-Marshall index would still have the highest value except for the Laspeyres index and the Fisher index would still have the lowest value except for the Paasche index. When the relative importance of year 0 compared to year 1 in volume terms is such as to make the Edgeworth-Marshall base prices look more like Laspeyres base prices, even a very high rate of inflation isn't going to change this.

High rates of inflation combined with a sharply declining volume of output is by no means an unrealistic scenario. This was the situation in virtually every country in the former Soviet Union for several years or more after 1991.

Second, the Laspeyres and Paasche base prices from which the Fisher volume measures are built are constructed like the Edgeworth-Marshall base prices, only for a single year instead of two. So if a high inflation rate is deemed to distort the weighting pattern of an Edgeworth-Marshall volume aggregate in favour of year 1 over year 0, by the same token it distorts the weighting pattern away from commodities that are purchased more in the first quarter and in favour of commodities that are purchased more in the fourth quarter. Therefore, in a Fisher aggregate under high inflation, Christmas trees and turkeys will tend to be overweighted, package holiday trips to winter sun spots underweighted.

Logically, if the Edgeworth-Marshall formula is considered inferior to the Fisher formula on this basis, the Fisher formula would be inferior to the formula:

$$Q_{1/0}^{F\,mod} = \left[\prod_{q=1}^{4} \left[\frac{\sum p_{0q} q_1}{\sum p_{0q} q_0} \frac{\sum p_{1q} q_1}{\sum p_{1q} q_0} \right]^{1/2} \right]^{1/4}$$

i.e. the geometric mean of the volume ratios for year 1 compared to base year 0 as weighted by each set of quarterly prices for the two year period. One could also envision a monthly version of the same formula.

However, it seems unproductive to let a high inflation rate dictate the choice of formula when Peter Hill (1996) has already proposed a workable solution in terms of constant price level (CPL) accounts, of dealing with the same problem, whatever index formula is used for volume series. It would involve the deflation of all value weights by the same general price index for the overall economy before being used in the index formula. Although the exact mechanics of his solution are open to debate, for certain some such method could be applied, and would be applicable to a volume index based on any formula.

It should be noted that where such a method was applied to a Fisher volume index, because it would use adjusted value weights, it would no longer satisfy the factor reversal property. Hill (1996; p.49) notes that "under high inflation, it is not possible to partition changes in the aggregate values in the current accounts into price and quantity changes both of which are acceptable as index numbers in their own right."

So while a high inflation environment would not strip the Edgeworth-Marshall formula of its superior representativeness compared to the Fisher formula, it would certainly strip the Fisher formula of its claim to satisfy the factor reversal property since with CPL accounts this would no longer be true.[20]

9. What Are the Remedies for Small or Developing Countries?

The SNA93 recommendations for price and volume measures do impose onerous statistical requirements on the agencies that would implement them, and the modifications to them suggested in this paper do not greatly reduce this burden. The SNA93 manual itself recognized that its preferred methodology, its A-level methodology, annual-link chain Fisher volume measures, was perhaps too costly to implement for many countries. It therefore also suggested a B-level methodology, annual-link chain Laspeyres volume measures, as an acceptable alternative.

The SNA93 A-level methodology has been poorly received by the international community; only the United States and Canada at the time of writing have implemented annual-link chain Fisher volume measures, and Canada only for its industry estimates. The SNA93 B-level methodology has been much better received. Eurostat has gone on to recommend annual-link chain Laspeyres countries for the European Community, and all its members have either

[20] The claim is inflated in any case, since due to the complications involved in deflating the value of physical change in inventories, factor reversal is not satisfied for such series in the Fisher world, nor for any aggregate containing VPC series, including total GDP itself.

converted to this methodology or are in the process of doing so. Other developed countries like Australia have also implemented SNA93's B-level methodology.

However, a number of countries, and not all of them in the developing world, essentially remain in the SNA68 universe, seeming to have neither the resources for chain Fisher or for chain Laspeyres estimates. I suspect they would not be persuaded to summon up the resources to calculate chain Edgeworth-Marshall estimates either.

For these countries there is probably no immediate escape from chain Laspeyres volume measures and their corresponding Paasche price indexes. But the SNA93 standards has left them with an all-or-nothing choice between annual chain linking and continuing as they are. So they will likely continue as they are, with Laspeyres volume series using constant prices for a base year 10 or 15 or more years in the past.

However, as discussed in section 4, such estimates could be greatly improved simply by replacing a ten-year rebasing cycle with a five-year rebasing cycle and making the base year the third or central year of the five-year span) rather than the initial year. In fact, in one important respect such estimates would be superior to those of many countries that have adopted annual-linked chain Laspeyres measures, since with less frequent links the series would be less prone to chain index drift.

And in addition to the corresponding Paasche price indexes, developing countries should be encouraged to calculate Laspeyres price indexes, which like the volume series would follow a five-year rebasing cycle with the basket reference year the central year of the five-year span.

While such a program would be considerably inferior to the system of annual-link chain Edgeworth-Marshall price and volume series outlined above, it would still mark a considerable advance over the SNA68 standards, and the index number procedures of most national accounting agencies in the developed world for most of the 20th century.

These volume measures might be comparable with direct Laspeyres volume aggregates calculated by developed countries, since according to SNA93 they are supposed to calculate these as analytical adjuncts to their annual-link chain measures.

10. Conclusion

The recommendations of this paper regarding chain indexes are fairly conventional as regards linking. I agree with the idea of chaining at the annual level and that is also what the SNA93 recommends and what most countries that calculate experimental or official chain volume series have done.

I also support the calculation of fixed-price volume aggregates rather than Fisher volume indexes. This conclusion, although at variance with SNA93 and with BEA and Statistics Canada practice, is in keeping with the decisions made by Eurostat and by the Australian Bureau of Statistics, and with the advice of a number of economists who have studied the subject.

However, most economists who have favoured chaining using fixed-price volume series have opted for Laspeyres aggregates, with only Erwin Diewert (1996), so far as I know, favouring the Walsh formula. No-one, to the best of my knowledge, has ever recommended the

Edgeworth-Marshall formula for use in the SNA. Statistics Sweden planned to revamp their consumer price index as a chain Edgeworth-Marshall index but finally they opted for the quite similar Walsh formula instead.

In recent years, the German economist Claude Hillinger has recommended using Edgeworth-Marshall price indexes as deflators to calculate volume aggregates.[21] It is not within the scope of this paper to comment on Professor Hillinger's work in detail, but it should be underlined that his proposal is quite different from the one in this paper. However, Professor Hillinger did a good thing at least in bringing renewed interest to the Edgeworth-Marshall formula.

This formula has always had its defenders, from Knibbs in the 1920's to Krtscha in the contemporary period, but it has never had as strong backing as other formulae. In my view this is due to an undue emphasis on whether formulae pass the factor reversal test and on whether they are exact for an aggregator function, neither of which the Edgeworth-Marshall formula does.[22] However, time reversal, matrix consistency (or additivity), the new good property and the property of transactions equality are important considerations too -- in my judgement, *more* important considerations -- for choosing a chain index formula, and the Edgeworth-Marshall formula is unique in possessing all of these properties.

The Walsh formula, which *is* exact for a utility function, is just as good with respect to time reversal, matrix consistency and the new good property, but fails the transactions equality property.

If I had only a 90-second TV slot to deliver my message, this would be it:

Chain price and volume aggregates with annual links hold the promise of improved measures of growth. It is most unfortunate that discussion has centred on two formulae, the Laspeyres and the Fisher, neither of which is well-suited for the calculation of chain aggregates. The Laspeyres formula doesn't pass the time reversal test, and a healthy fear of chain drift should lead us to reject it. The Fisher formula passes the time reversal test, but it fails the matrix consistency test, is not even weakly consistent in aggregation and fails the new good test, all of which make it an awkward and unsatisfying formula for both producers and consumers of National Accounts estimates. None of these criticisms hold for the Edgeworth-Marshall formula and only the Edgeworth-Marshall formula has the property of transactions equality that has very important implications for growth measurement in the new economy. So far as any one formula will be the formula for price and volume measurement in the National Accounts in the 21st century, it should be the Edgeworth-Marshall formula. However, we should cease to try to make all industries or commodities fit into the Procrustean bed of one formula and, where required, we should change

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[21] See Hillinger (2000). A number of papers have been written that comment on his methodology, one of which -- Ehemann et al. (2000) -- is listed in the references to this paper.

[22] The true factorial price index that is consistent with a Leontief, fixed-coefficients utility function is the ratio of an expenditure index to an Edgeworth-Marshall quantity index as shown by Balk (1983). This is *not* the Edgeworth-Marshall implicit price index defined by (4.6): its formula would be:

$$(\sum p_1 q_1 / \sum p_0 q_0)/(\sum p_{01} q_1 / \sum p_{01} q_0).$$

But the Edgeworth-Marshall price index itself is not consistent with this or any other utility function, and in any case, a "superlative" index that is only consistent with a fixed-coefficients utility function is of very limited interest.

our formulae for price and volume measures to take account of cyclical and seasonal commodities. Finally, matrix consistency is dependent not only on formula choice but on linking policy. We must follow the good example of Australia and the United Kingdom, and change our reference year annually, linking our chain-volume estimates backward rather than forward.

Appendix: Weak, Specific and Strong Consistency in Aggregation

So far in the literature on index numbers, the discussion of consistency in aggregation has revolved around price rather than volume indexes. For now, let us then stick to definitions of consistency in aggregation in the domain of price indexes.

Balk (1996) [23]defines the following properties for an index to be consistent in aggregation:

1. the index for the aggregate, which is defined as a single-stage index, can also be computed in two stages, namely by first computing the indexes for the subaggregates and from these the index for the aggregate;

2. the indexes used in the single-stage computation and those used in the first stage computation have the same functional form,

3. the formula used in the second stage computation has the same functional form as the indexes used in the single and in the first stage after the following transformation has been applied: elemental indexes are replaced by subaggregate indexes and the values of the elemental indexes are replaced by subaggregate values.

Balk takes an all-or-nothing approach to consistency in aggregation, and would not consider a formula that satisfies the first two properties to be consistent in aggregation, although he notes that Blackorby and Primont (1980) ignored the third property altogether in their definition of consistency in aggregation. It probably makes better sense to consider an index formula that meets all three criteria as being strongly consistent in aggregation, and one that meets only the first two criteria as being weakly consistent in aggregation.

Balk notes that the Walsh formula, which is very similar to the Edgeworth-Marshall one, satisfies the first two criteria for a price index but not the third. This is also true of the Edgeworth-Marshall price index.

Auer (2004) writes of weak and specific consistency in aggregation; specific consistency in aggregation, as he defines it, is identical with Balk's criteria.[24] However, he notes that the Edgeworth-Marshall price index can be defined in terms of base and comparison period values, but also in two alternative ways involving values at constant prices, and for these definitions, single-stage and two-stage solutions are identical. It therefore passes the weak consistency test.

[23] See Balk (1990, p. 358-359). I have largely adopted Balk's phrasing.

[24] Auer (2004) also discusses strict consistency in aggregation, which would imply that consistency in aggregation holds whichever variant of the index formula is used to define it. Although the Stuvel and Montgomery-Vartia price formulae are specifically consistent in aggregation, neither is strictly consistent in aggregation.

Although there is a difference between weak consistency as defined by Balk's first two properties and weak consistency in Auer's sense, the Edgeworth-Marshall formula is weakly consistent by either definition, as is the Walsh formula.

The Fisher formula is not even weakly consistent in aggregation, and one cannot (or at least should not) calculate a Fisher of Fishers. At any stage of aggregation one must have a set of Laspeyres and Paasche indexes and the Fisher index should be calculated as their geometric mean. Auer also shows that if one erroneously does treat the Fisher formula as being consistent in aggregation, the empirical implications of the error can be far from trivial. In his example, calculating a Fisher of Fishers changes a correctly estimated 1.6% price increase into a misestimated price decrease of 0.5%.[25]

He is also surely right in believing that the important distinction is between formulae that are weakly consistent in aggregation (like Edgeworth-Marshall) and not consistent at all (like Fisher). Strong consistency (or specific consistency), by contrast, is not particularly important.

All of the formulae that are strongly consistent in aggregation without exception have one or more very bad properties that would seem to more than offset any advantage this might incur over their weakly consistent rivals. Neither the Laspeyres formula nor the Paasche formula satisfies the time reversal property; neither the Montgomery-Vartia nor the Stuvel formula satisfies proportionality.

Expressed in terms of value series and price deflators, the Edgeworth-Marshall volume index can be calculated as:

$$\text{(A1)} \quad \frac{\sum p_{01}q_1}{\sum p_{01}q_0} = \frac{\sum \frac{v_0 + v_1}{q_0 + q_1} q_1}{\sum \frac{v_0 + v_1}{q_0 + q_1} q_0} = \frac{\sum \frac{v_0 + v_1}{p_0 q_0 + p_0 q_1} p_0 q_1}{\sum \frac{v_0 + v_1}{p_0 q_0 + p_0 q_1} p_0 q_0} = \frac{\sum \frac{v_0 + v_1}{v_0 + v_1 / P_{1/0}} v_1 / P_{1/0}}{\sum \frac{v_0 + v_1}{v_0 + v_1 / P_{1/0}} v_0}$$

or as

$$\text{(A2)} \quad \frac{\sum p_{01}q_1}{\sum p_{01}q_0} = \frac{\sum \frac{v_0 + v_1}{q_0 + q_1} q_1}{\sum \frac{v_0 + v_1}{q_0 + q_1} q_0} = \frac{\sum \frac{v_0 + v_1}{p_1 q_0 + p_1 q_1} p_1 q_1}{\sum \frac{v_0 + v_1}{p_1 q_0 + p_1 q_1} p_1 q_0} = \frac{\sum \frac{v_0 + v_1}{v_0 P_{1/0} + v_1} v_1}{\sum \frac{v_0 + v_1}{v_0 P_{1/0} + v_1} v_0 P_{1/0}}$$

Using the formulation on the rightmost side of either (A1) or (A2), the Edgeworth-Marshall volume index gives the same result for Auer's example of a three-commodity economy whether calculated in two steps or in a single step.

For statistical agencies themselves, the failure of a formula to satisfy either weak or strong consistency in aggregation is probably more of a nuisance issue than a real issue. The problem is more what users who are not well-versed in index number properties will do with data series. In this respect, data users are more likely to go seriously wrong working with Fisher aggregates than they are with Edgeworth-Marshall ones.

[25] See Auer (2004), pp.385-386.

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